The Role of Formal Semantics in Linguistics

October 22, 2012

Abstract

We explain what role formal semantics does and should play for an empirical linguist. By looking at a modified non-modal version of PL1 that allows for representing certain types of contextual dependence two relevant roles are illustrated. In the end we argue that in analogy with other empirical and formal disciplines the technical apparatus should be a framework of abstraction for the semantics of natural languages.

1 The Modus Operandi of Formal Semantics

Which roles does, can and should formal semantics play for an empirical linguist? Which role does and can the investigation of the semantics of natural languages by means of formal mathematical methods play for an empirical linguist? Which role should it play? In formal semantics, natural language sentences are being formalized and through this the natural language semantics can be represented by the semantics of the formal language. Hence, we have the following ingredients:

- A semantics for the natural language
- A semantics for the formal language
- A translation scheme between the natural and the formal language.

Keeping the translation fixed, we have $2 \cdot 2$ possibilities of how the interest in these two semantic theories can be combined:

1. Interest in formal semantics without interest in applications to natural language semantics (e.g. as applied in mathematics and computer sciences).

2. Interest in formal semantics with interest in applications to natural language semantics (e.g. formalization/regimentation and analysis of the relevant natural language sentence in the formal semantics).

3. Interest in natural language semantics without interest in applications to formal semantics.

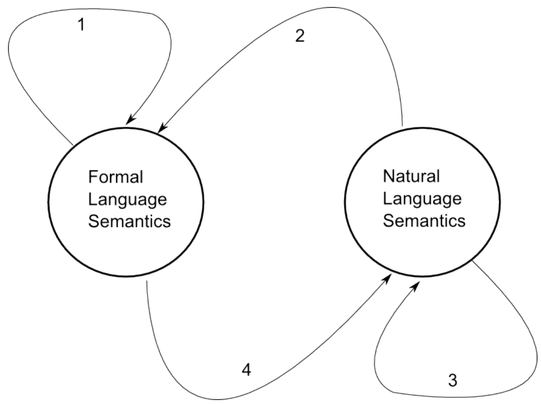

Figure 1: Combinatorics of interest in formal and natural language semantics. The edges point to the field of interest and start at the field that motivates the research.

4. Interest in natural language semantics with interest in applications to formal semantics (e.g. development of new formal semantic theories motivated by linguistic phenomena that cannot properly be explained by existing formal theories).

Figure 1 demonstrates the four kinds of constellations. 1:
In what follows, we will illustrate the various options using an example of a model of formal semantics that we have developed. We will summarise our arguments by briefly explaining why option 3 is not reasonable.

2 Application to Natural Languages

2.1 Donkey sentences

There are English sentences which are difficult to formalize in first-order logic without giving up regularity in translation. One of the best examples to illustrate it is the infamous “Donkey sentence”: (D) “every farmer who owns a donkey beats it”. The first-order translation of (D) that gives the right truth conditions appears to be (D’) “$\forall xy\,(Fx \wedge Dy \wedge Oxy \rightarrow Bxy)$”. However, this shows that we cannot give a unified treatment of the translation of indefinites. The problem is that in (D), the indefinite “a donkey” seems to have “universal force” ([**?**]) and needs to be translated using a universal quantifier which has wide scope, whereas in many other cases indefinites will be correctly translated using existential quantifiers with narrow scope (consider: “A man walks by”). Can we provide a translation of sentences containing anaphora as in (D), and an interpretation, which preserves regularity in translation? In particular: can we provide a translation of (D) in which we use the existential quantifier for “a donkey”? Indeed, the “most natural” translation of (D) appears to be (D”)

"$\forall x\,((Fx \wedge \exists y\,(Dy \wedge Oxy)) \rightarrow Bxy)$". Unfortunately, (D") does not deliver the right truth conditions when classically interpreted. Can we solve this problem by modifying the way we interpret first-order formulas? This is an example for an investigation of types 2 and 4.

We argue that the problem admits of a solution in a broadly model-theoretic framework. In particular, we sketch a route to modify and extend classical first-order model theory which incorporates Kit Fine's (2007) idea of coordination (= type 4 investigation), and show how this approach can be extended to sequences of formulas (= type 1 investigation). Using coordination, we can provide an interpretation such that the translation (D")

$$\forall x\,((Fx \wedge \exists y\,(Dy \wedge Oxy)) \rightarrow Bxy)$$

comes out correct (= type 2 investigation). The idea is to add the information that the free occurrence of y is appropriately "coordinated" with its bound occurrences, which implies that the free occurrence of y is to be interpreted with the value that renders "$Dy \wedge Oxy$" true. To be more precise, consider (D") with the occurrences of variables made explicit by counting them (D"Num):

$$\forall x_1\,((Fx_2 \wedge \exists y_1\,(Dy_2 \wedge Ox_3y_3)) \rightarrow Bx_4y_4)$$

We now add a "coordination constraint", namely that y_4 is always to have the same semantic value as y_2 and y_3. Thus, if the antecedent of (D"Num) has an interpretation that renders it true, and the conditional is not vacuously true, then in order to evaluate the truth value of the conditional, y_4 must be interpreted accordingly. If such constraints can be incorporated into our semantics, a solution of the problem of Donkey pronouns is in sight.

To put it simply, we enforce the coordination constraint by evaluating (D"Num) as if it was in prenex normal form, whereas a coordination scheme tells us that the free occurence of y (y_4) is not to be renamed, and thus interpreted in the same way as the other occurences of y):

$$\forall x_1 \forall y_1\,((Fx_2 \wedge Dy_2 \wedge Ox_3y_3) \rightarrow Bx_4y_4)$$

It is immediate that this yields the intended interpretation.

2.2 The proposal in a nutshell

In our semantics, coordination is nothing else than a partition on occurrences of variables. A *coordination scheme* for a variable v_i and a formula ϕ is a partition of occurrences of the variables in ϕ. A *valid coordination scheme* for the variable v_i and formula ϕ is a coordination scheme for v_i and ϕ with the following property:

- All variable occurrences bound by a quantifier (including the variable following the quantifier) are in the same partition.

A *coordination scheme* for ϕ is a list of coordination schemes for all variables that occur in ϕ. A *valid coordination scheme* for ϕ is a coordination scheme for ϕ with all its members being valid coordination schemes for ϕ and the respective variable.

The idea is to interpret formulas not only with respect to a model and an assignment, but with respect to a coordinated assignment — and assignment to occurrences that respects a valid coordination scheme: all occurrences in the same partition are to be assigned the same values.

However, quantification only works when formulas are in prenex normal form: for a valid initial partition could coordinate a free variable occurrence with bound occurrences somewhere else in the formula (as in the Donkey sentence). We need to build this into our semantics.

To this end, we use a translation function $N(\phi)$ that maps ϕ to its prenex normal form, ignoring restrictions that have to do with free occurrences of variables that would become bound by the formation rules (we can ignore this because the *valid* coordination scheme and the coordinated assignment takes care of such cases. For example, we would translate $\forall v_1 (Pv_1) \wedge Qv_1$ to $\forall v_1 (Pv_1 \wedge Qv_1)$. If all occurrences of x are coordinated, this translation is correct anyway. If the last occurrence is not coordinated with the bound occurrences, the coordination scheme will not coordinate the assignment of values to the first occurrences and the last occurrence of x. It is well known how to translate formulas to prenex normal form, so we regard $N(\phi)$ as sufficiently clear.

How do coordinated assignments work in more detail?

A *occurrence assignment* β for ϕ is an assignment to all occurrences of all variables in ϕ. A *coordinated occurrence assignment* β_C for a formula ϕ and a valid coordination scheme C is an occurrence assignment with the following property (v_{ij} denotes the j-th occurrence of the i-th variable, and C_i is the coordination scheme for the i-th variable in C):

- If $\exists k (\beta_C(v_{ik})$ is defined and $\exists p \in C_i (v_{ik} \in p \wedge v_{ij} \in p))$, then $\beta_C(v_{ij}) = \beta_C(v_{ik})$

So, for example, if the first three occurrences of v_1 are coordinated (i.e. if they are in the same partition), all three occurrences get assigned the value that one of the occurrences gets assigned (we need to make sure, of course, that only one occurrence gets assigned a value, and the values of all other occurences are undefined, so that there is no conflict). Thus, if all occurrences of v_i are coordinated, it suffices to fix a value of one of its occurrences.

This yields a way to handle quantification: since all bound occurrences are coordinated, it suffices to determine β such that it assigns a value to the occurrence of the variable following the quantifier.

We can now define the notion of "a model and a coordinated occurrence assignment satisfying a formula" (note that we assume that the number of occurrence does not change if we consider a subformula. For example, the number of occurrence of x in "$P(x)$" is 2 , if we come from the formula "$Q(x) \wedge P(x)$"):

- $M, \beta_C \models \phi \Leftrightarrow M, \beta_C \models N(\phi)$, if ϕ is not in prenex normal form
- $M, \beta_C \models \forall x_{ij}\phi \Leftrightarrow \forall a\,(M, \beta_C\,(x_{ij} : a) \models \phi)$
- $M, \beta_C \models \exists x_{ij}\phi \Leftrightarrow \exists a\,(M, \beta_C\,(x_{ij} : a) \models \phi)$
- $M, \beta_C \models \phi \wedge \psi \Leftrightarrow M, \beta_C \models \phi$ and $M, \beta_C \models \psi$ (if $\phi \wedge \psi$ is quantifier free)
- $M, \beta_C \models \phi \rightarrow \psi \Leftrightarrow$if $M, \beta_C \models \phi$, then $M, \beta_C \models \psi$ (if $\phi \rightarrow \psi$ is quantifier free)
- $M, \beta_C \models \neg\phi \Leftrightarrow$, if not $M, \beta_C \models \phi$ (if $\neg\phi$ is quantifier free)
- $M, \beta_C \models Pv_{ij} \Leftrightarrow \beta_C\,(v_{ij}) \in P^M$
- $M, \beta_C \models v_{ij} = v_{lm} \Leftrightarrow \beta_C\,(v_{ij}) = \beta_C\,(v_{lm})$

We now provide some examples to render it clear how the semantics works.

2.2.1 Evaluating the Donkey Sentence

The relevant translation of the donkey sentence is (D"):

$$\forall v_1\,((Fv_1 \wedge \exists v_2\,(Dv_2 \wedge Ov_1v_2)) \rightarrow Bv_1v_2)$$

Of course, the last occurrence of v_2 needs to be coordinated with its other occurrences. Since all other v_1 and v_2 are coordinated because of the quantifiers, C looks as follows:

$$C = \{\{\{v_{1,1}, v_{1,2}, v_{1,3}, v_{1,4}\}\}, \{\{v_{2,1}, v_{2,2}, v_{2,3}, v_{2,4}\}\}\}$$

We need to show that our interpretation of (D") under C in "classical coordination semantics" is equivalent to the classical interpretation of (D'):

$$\forall v_1 v_2\,(Fv_1 \wedge Dv_2 \wedge Ov_1v_2 \rightarrow Bv_1v_2)$$

Donkey-Sentence$M, \beta_C \models D'' \Leftrightarrow M, \beta \models_{Class} D$ We simply evaluate the formula (we omit the information about the number of occurences for readability):

$$\begin{array}{cc}
M, \beta_C \models \forall v_1\,((Fv_1 \wedge \exists v_2\,(Dv_2 \wedge Ov_1v_2)) \rightarrow Bv_1v_2) & \Leftrightarrow \\
M, \beta_C \models \forall v_1 \forall v_2\,(Fv_1 \wedge Dv_2 \wedge Ov_1v_2 \rightarrow Bv_1v_2) & \Leftrightarrow \\
\forall a \in M\,(M, \beta_C\,(v_{1,1} : a) \models \forall v_2\,(Fv_1 \wedge Dv_2 \wedge Ov_1v_2 \rightarrow Bv_1v_2)) & \Leftrightarrow \\
\forall ab \in M\,(M, \beta_C\,(v_{1,1} : a, v_{2,1} : b) \models Fv_1 \wedge Dv_2 \wedge Ov_1v_2 \rightarrow Bv_1v_2) & \Leftrightarrow \\
\forall ab \in M\,(M, \beta\,(v_1 : a, v_2 : b) \models_{Class} Fv_1 \wedge Dv_2 \wedge Ov_1v_2 \rightarrow Bv_1v_2) & \Leftrightarrow \\
M, \beta \models_{Class} \forall v_1 v_2\,(Fv_1 \wedge Dv_2 \wedge Ov_1v_2 \rightarrow Bv_1v_2) &
\end{array}$$

(Note that we obtain the second last line because of the definition of β_C.)

3 Application to Formal Languages

There is a possibility to translate coordinated sentences to equivalent uncoordinated sentences (which can then be evaluated by classical semantics). The process of translation is very similar to the process of evaluating coordinated formulas presented above. This question is of type 1.

3.0.2 The algorithm

We can translate every coordinated sentence into a first-order language, and evaluate it classically. Basically, we emulate uncoordinated variables of the same name by using different variable names, and converting the formula to prenex normal form (ignoring restrictions for free variables that become bound by applying the formation rules).

Step 1 We start with a sentence S and a valid initial coordination scheme C. We first need to emulate occurrences of uncoordinated variables by using different variable names. To this end, we define a function $\pi_S(x_{n,i}, C)$, which assigns appropriate variables to the ith occurrence of the nth variable of the C-coordinated sentence S (where $NV(S)$ is the total number of occurrences of variables in S):

$$\pi(x_{n,i}, C) = \begin{cases} x_{(NV(S)\cdot(n-1))+j}, \text{ if} & \exists j\,(\{x_{n,j}, x_{n,i}\} \in C \wedge j \leq i) \\ x_{(NV(S)\cdot(n-1))+i} \text{ otherwise} & \end{cases}$$

The first translation step T_2 just replaces the i-th occurrence of the n-th variable by the above-defined $\pi(x_{n,i}, C)$:

$$T_2(S, C) = S[x_{n,i}/\pi(x_{n,i}, C)], \text{ for all } n, i.$$

Since C is a valid coordination scheme, all occurrences of variables bound by the same quantifier get assigned the same variable. Moreover, variables coordinated with bound variables get assigned the same variable name, even if they are not bound by the quantifier.

Step 2 We convert S to prenex normal form, using the well-known conversion rules, ignoring restrictions of the conversion rules for free variables (we want to bind variables which are coordinated and thus got assigned the same name).

3.0.3 Examples

The donkey sentence The relevant translation of the donkey sentence is (D”), with $x = v_1$ and $y = v_2$:

$$\forall v_1\,((Fv_1 \wedge \exists v_2\,(Dv_2 \wedge Ov_1v_2)) \rightarrow Bv_1v_2)$$

All four occurrences of v_2 are coordinated. Thus:

$$C = \{\{\{v_{1,1}, v_{1,2}, v_{1,3}, v_{1,4}\}\}, \{\{v_{2,1}, v_{2,2}, v_{2,3}, v_{2,4}\}\}\}$$

The application of the function π will not do anything important, since all variables are coordinated. However, because of its particular definition, it will still change the variable names. In particular, since $NV(S) = 8$, occurrences of of v_2 will become occurrences of v_8.

What remains to be done is to convert the formula to prenex normal form, ignoring restrictions for binding free variables. In this way, we obtain the following:

$$\forall v_1 ((Fv_1 \wedge \exists v_8 (Dv_8 \wedge Ov_1v_8)) \rightarrow Bv_1v_8) \quad (1)$$
$$\forall v_1 (\exists v_8 (Fv_1 \wedge Dv_8 \wedge Ov_1v_8) \rightarrow Bv_1v_8) \quad (2)$$
$$\forall v_1 \forall v_8 ((Fv_1 \wedge Dv_8 \wedge Ov_1v_8) \rightarrow Bv_1v_8) \quad (3)$$

This is a correct classical translation of the Donkey sentence.

Free variables Consider the formula (F):

$$v_1 = v_1 \rightarrow v_2 = v_2$$

If there is no coordination whatsoever, the translation of (F) will be:

$$v_1 = v_2 \rightarrow v_5 = v_6$$

This formula is not a logical truth. If the occurrences of v_1 and v_2 are coordinated, however, the translation of (F) will be:

$$v_1 = v_1 \rightarrow v_5 = v_5$$

This is a logical truth.
In the last sections we saw examples for the different types of interest in a semantics as demonstrated in Fig. 1. We saw how closely these options interact. It is always a package deal, if you take one type you take the other two types as well.

4 Why Formal Work Matters in Linguistics

So far we have seen which roles formal semantics does and can play in linguistics. But we have not touched the question which role formal semantics should play. If a goal of science is to find out the truth, then the ultimate goal of semantics is to

- give the "true" semantics of natural language (call this *Universal Semantics*).

A formal linguist will hope that this true semantics of natural language can eventually be represented formally as well.

However, this has not been done as yet.

The question is whether this is a problem for the application of formal methods to natural languages. Aren't there way too many phenomena which can't be adequately described by the available formal theories? Does this mean that all our formal theories are wrong?

The answer is, first, that the ones who reject working formally haven't found the true semantics as well. Second, the formal representation should be - as is common in other formal and empirical disciplines - viewed as an abstraction. Not all formal theories are on the same level (Should one use first-order, second-order predicate-logic, first- or second- order modal logic?), some are richer in expressive power some are smoother to handle. Depending on which particular goal we have in the investigation of the empirical phenomenon of a natural language, we choose a particular theory and thereby a particular level of abstraction for our description. The above discussed semantics, for instance, can be used as a description of context-dependence without leaving the familiar classical model-theory. But there are limits to any abstraction, i.e. there are always phenomena that cannot be properly explained by the abstraction.

And to know the limits of his abstraction is necessary for any user. If an electrical engineer describes integrated circuits with Kirchhoff's laws, he has to be clear about the fact that these laws only hold if at any instant

1. through every path inside the element $\frac{\partial \Phi_B}{\partial t} = 0$, where Φ_B is the magnetic flux, and
2. $\frac{\partial q}{\partial t} = 0$, where q is the whole charge inside the element, and
3. $d \ll \lambda$, where d is the area of the circuit and λ the wavelength of the relevant signal.

If these conditions are not met, Kirchhoffs Laws cannot be applied. Instead more complicated equations like Maxwell's must be taken for a proper explanation of the behaviour of the circuit. Likewise, formal semantics is just one possibility to describe the properties of natural languages at various levels but it is certainly not the only one when one wants to engage in semantics. So the role of formal semantics is:

- Give an empirically adequate description of natural language fragments. Use this description for purposes where intuitions generally blur, and data scatter, like the syntax-semantics interface, the pragmatics-semantics interface, etc.

Formal work distinguishes itself by being extremely robust. And this does not only concern the semantics of natural languages but seems to be a basic property of our cognitive apparatus. Here is an example inspired by Kahnemann (2011).

A car and a car-jack cost together 350€. The car is 30000€ more expensive than the car-jack. How much is the car-jack?

Most people answer 500€. This answer ist wrong. If one expresses the problem via the equations $x = y + 30000$ and $30500 = x + y$ then one has two equations for two unknowns and thus a solution, which is 250€.

An example for the advantage of robust reasoning in linguistics is the semantics-pragmatics interface. Pragmatic content is often propositional. So how can we distinguish semantic and pragmatic content systematically? Consider the old Gricean example: I put off my shoes and went into bed. The formalization is $p \wedge q$. This sentence is equvalent (and by construction of the semantics of propositional logic therefore als synonymous to) $q \wedge p$. The chronological component, that is communicated by the order of appearance in the conjunction vanishes. Hence, we know that this is a pragmatic effect. The propositional, pragmatic content "First I went into bed and then I put off my shoes" is therefore different from the semantic one. Propositional logic constitutes a nice example of how we can robustly control the semantics of a sentence. This suggests distinguishing at least two kinds of pragmatic "and" conjuncts depending for instance on the ontological type of the flanking constituents. One induces a chronological order, the other doesn't.

5 Acknowledgments

I want to warmly express my gratitude to Ali Rustaee und Adam Golebiowski. Sections 2 and 3 are *joint* work with my friend Alexander Oldemeier, University of Leeds. It is his work to the same extent as it is mine. Furthermore he was very generous in very helpfully commeting on this paper. Thanks Karl-Georg Niebergall for staring my way from behind the curtains.

References

[1] Fine, K. 2007. *Semantic Relationism*. Oxford: Blackwell.

[2] Geurts, Bart and Beaver, David I., "Discourse Representation Theory", The Stanford Encyclopedia of Philosophy (Fall 2011 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/fall2011/entries/discourse-representation-theory/>.

[3] Kahnemann, D. 2011. Thinking, Fast and Slow. New York: Macmillan.